Int. MBA(HA) 4th Semester

Code- BHI-404 Fundamentals of Financial Management

4th Unit (A) Time Value of Money

The concept that money available today is worth more than the same amount of money in the future.

This preference rests on the Time value of money.

Thus, a money received today is worth more than a money received tomorrow, why?

This is because that (a) a money received today can be invested to earn interest (b) Due to money's potential to grow in value over time. (c) Because of this potential, money that's available in the present is considered more valuable than the same amount in the future (d) Time Value of Money is dependent not only on the time interval being considered but also the rate of discount used in calculating current or future values. (e) Based on this, we can use the time value of money concept to calculate how much you need to invest or borrow now to meet a certain future goal

You need to sell your Bicycle and you receive offers from three different buyers.

The first offer gives Rs. 3,000 to be paid now

The second offer gives Rs. 3,100 to be paid one year from now

The third offer gives Rs. 3,600 Birr to be paid after five years.

> Assume that the second and third offers have no credit risk. The risk free interest rate is 5%. Which offer would you accept?

Interest

A rate which is charged or paid for the use of money. An interest rate is often expressed as an annual percentage of the principal.

a nominal interest rate, is one where the effects of inflation have not been accounted for.

Real interest rates are interest rates where inflation has been accounted for

Changes in the nominal interest rate often move with changes in the inflation rate, as lenders not only have to be compensated for delaying their consumption, they also must be compensated for the fact that a dollar will not buy as much a year from now as it does today.

What are the components of interest rate?

Real Risk-Free Rate - This assumes no risk or uncertainty, simply reflecting differences in timing.

Expected Inflation - The market expects aggregate prices to rise, and the currency's purchasing power is reduced by a rate known as the inflation rate.

Default-Risk Premium - What is the chance that the borrower won't make payments on time, or will be unable to pay what is owed? This component will be high or low depending on the creditworthiness of the person or entity involved.

Liquidity Premium- Some investments are highly liquid, meaning they are easily exchanged for cash. Other securities are less liquid, and there may be a certain loss expected if it's an issue that trades infrequently.

Holding other factors equal, a less liquid security must compensate the holder by offering a higher interest rate.

Maturity Premium - Other things being equal, a bond obligation will be more sensitive to interest rate fluctuations, if maturity period is longer.

Simple Interest and Compound Interest

When you deposit money into a bank, the bank pays you interest.

When you borrow money from a bank, you pay interest to the bank.

Simple interest is money paid only on the principal

$I = P \bullet r \bullet t$

Whers as

Rate of interest is the percent charged or earned.

Principal is the amount of money borrowed or invested.

Time that the money is borrowed or invested (in years).

Compound interest

Compound interest is interest paid not only on the principal, but also on the interest that has already been earned. The formula for compound interest is below.

$$A = p(1 + r)^t$$

"A" is the final dollar value, **"P"** is the principal, **"r"** is the rate of interest, and **"t"** is the number of compounding periods per year.

Uses of Time Value of Money

Time Value of Money, or TVM, is a concept that is used in all aspects of finance including:

- Bond valuation
- Stock valuation
- Accept/reject decisions for project management
- Financial analysis of firms
- And many others!

Types of TVM Calculations

- There are many types of TVM calculations
- The basic types that will be covered are:
    - Future value of a lump sum
    - Present value of a lump sum
    - Present and future value of cash flow stream
    - Present and future value of annuities

Basic Rules:

The following are simple rules that you should always use no matter what type of TVM problem you are trying to solve:

1. Stop and think: Make sure you understand what the problem is asking.
2. Draw a representative timeline and label the cash flows and time periods appropriately.
3. Write out the complete formula using symbols first and then substitute the actual numbers to solve.
4. Check your answers using a calculator.

Example:

How much money will you have in 5 years if you invest Rs.100 today at a 10% rate of return?

Draw a timeline

- Write out the formula using symbols:

FVt = CF0 * (1+r)t

The Future value of Money

Future value is the value of an asset at a specific date in the future.

It measures the nominal future sum of money that a given sum of money is "worth" at a specified time in the future assuming a certain interest rate, or more generally, rate of return.

Actually, the future value does not include corrections for inflation or other factors that affect the true value of money in the future.

Future Value Equation

- $FVn = PV(1 + i)^n$
  - FV = the future value of the investment at the end of n year

- i = the annual interest rate
- PV = the present value, in today's dollars, of a sum of money

This equation is used to determine the value of an investment at some point in the future.

Future Value of a Lump Sum

Future value determines the amount that a sum of money invested today will grow to in a given period of time

You can think of future value as the opposite of present value

The process of finding a future value is called "compounding"

Example:

How much money will you have in 5 years if you invest Rs. 100 today at a 10% rate of return?

1. Draw a timeline
2. Write out the formula using symbols:

   $FV_t = CF_0 * (1+r)^t$

3. Substitute the numbers into the formula:

   $FV = Rs.100 * (1+.1)^5$

4. Solve for the future value:

   FV = Rs.161.05

Future Value of a Cash Flow Stream

The future value of a cash flow stream is equal to the sum of the future values of the individual cash flows.

The FV of a cash flow stream can also be found by taking the PV of that same stream and finding the FV of that lump sum using the appropriate rate of return for the appropriate number of periods.

The following equation can be used to find the Future Value of a Cash Flow Stream at the end of year t.

$$FV_n = \sum_{t=0}^{n} CF_t (1+r)^{n-t}$$

where

FVt = the Future Value of the Cash Flow Stream at the end of year t,

CFt = the cash flow which occurs at the end of year t,

r = the discount rate,

t = the year, which ranges from zero to n, and

n = the last year in which a cash flow occurs

Exercise

Find the Future Value at the end of year 4 of the following cash flow stream given that the interest rate is 10%.

Year	0	1	2	3	4
Cash Flow		100	200	200	300

Solution:

$$FV_4 = 100(1+.10)^3 + 200(1+.10)^2 + 200(1+.10)^1 + 300 = \$895.10$$

Annuities

An annuity is a cash flow stream in which the cash flows are all equal and occur at regular intervals.

To considered as annuity the following conditions must be present

The periodic payment must be equal in amount

The time period between payments should be constant

The interest rate per year remains constant

The interest is compounded at the end of each time period

Types of Annuities

There are different types of annuities, among these are

1. An *ordinary annuity* is one where the payments are made at the *end* of each period
2. An *annuity due* is one where the payments are made at the *beginning* of each period
3. A *deferred annuity* is one where the payments do not commence until period of times have elapsed
4. A *perpetuity* is an annuity in which the payments continue indefinitely

Ordinary Annuity

The amount of an ordinary annuity (annuity in arrears) consists of the sum of the equal periodic payments and compounded interest on the payments immediately after the final payments.

The *future value* (or *accumulated value*) of an annuity is the amount due at the end of the term

FV of ordinary annuity, it is the sum of all the periodic payments made and interest accrued up to and including the final payment period

$$\mathbf{FV_{ordianryAn\,nuity}} = \mathbf{CF}\left[\frac{(\mathbf{1}+\mathbf{r})^{\mathbf{n}}-\mathbf{1}}{\mathbf{r}}\right]$$

Where

CF = Cash flow per period

r = interest rate

n = number of payments

$FV_A = PMT(ACF)_{n,r}$ Using Table

The formula for future value

$$FVA_n = A\left[\frac{(1+r)^n - 1}{r}\right]$$

Example: What amount will accumulate if we deposit $5,000 at the **end** of each year for the next 5 years? Assume an interest of 6% compounded annually

PV = 5,000
i = .06

Year	1	2	3	4	5
Begin	0	5,000.00	10,300	15918.00	21,873.08
Interest	0	300	618	955.08	1,312.38
Deposit	5,000.00	5,000.00	5,000.00	5,000.00	5,000.00
End	5,000.00	10,300.00	15,918.00	21,873.00	28,185.46

$$\mathbf{FV_{ordianryAn\ nuity} = CF\left[\frac{(1+r)^n - 1}{r}\right]}$$

$$FV_{oa} = 5000\left[\frac{(1+0.06)^5 - 1}{0.06}\right]$$

$$FV_{oa} = 5000\left[\frac{(1.06)^5 - 1}{0.06}\right]$$

$$FV_{oa} = 5000\left[\frac{1.3382255776 \quad - 1}{0.06}\right]$$

$$FV_{oa} = 5000\left[\frac{0.3382255776}{0.06}\right]$$

$$FV_{oa} = 5000\ [5.63709296\ ]$$

$$FV_{oa} = 28185\ .4648$$

Annuity Due

A form of annuity where periodic receipts or payments are made at the beginning of the period and one period of the annuity term remains after the last payment.

$$FV_{\text{Annuity Due}} = C * \left[\frac{(1+i)^n - 1}{i}\right] * (1+i)$$

The **Future Value of an Annuity Due** is identical to an ordinary annuity except that each payment occurs at the beginning of a period rather than at the end.

Since each payment occurs one period earlier, we can calculate the present value of an **ordinary annuity** and then multiply the result by (1 + i).

$$FV_{ad} = FV_{oa}(1+i)$$

Where:

FVad = Future Value of an Annuity Due

FVoa = Future Value of an Ordinary Annuity

i = Interest Rate Per Period

Future value of Deferred Annuity

When the amount of an annuity remains on deposit for a number of periods beyond the final payment, the arrangement is known as a deferred annuity.

When the amount of an ordinary annuity continues to earn interest for an additional one year period we have an annuity due situation

When the amount of an ordinary annuity continues to earn interest for more than one additional periods, we have a deferred annuity situation.

Application

The future value annuity formula can be applied in different contexts.

Its important applications are

To know how much we have in the future

FV= $A(1+r)^n$

To know how much to save annually

$$A = FV \frac{r}{(1+r)^n - 1}$$

To find out the interest rate

$$i = \sqrt[n]{FV}$$

To know how long should wait to get the accumulated money

$$\log(1+r)^n = \log\frac{FV}{A}r$$

The Present Value of Money

- It is a process of discounting the future value of money to obtain its value at zero time period (at present)
- Present values tell you the amount you must invest today to accumulate a certain amount at some future time
- To determine present values, we need to know:
  - The amount of money to be received in the future
  - The interest rate to be earned on the deposit
  - The number of years the money will be invested

Discounting and Compounding

The mechanism for factoring in the present value of money element is the discount rate.

The process of finding the equivalent value today of a future cash flow is known as **discounting**.

Compounding converts present cash flows into future cash flows.

Calculating the Present Value

- So far, we have seen how to calculate the future value of an investment
- But we can turn this around to find the amount that needs to be invested to achieve some desired future value:
- Using the Present Value Table
- Present value interest factor (PVIF):
  a factor multiplied by a future value to determine the present value of that amount (PV = FV(PVIFA)
- Notice that PVIF is lower as the number of years increases and as the interest rate increases
- It can also be calculated using a financial calculator

  $PV = [FV/(1+r)^n]$

Cash Flow Types and Discounting Mechanics

There are five types of cash flows -

- single cash flows (Lump sum cash flows),
- Cash flows stream
- annuities,
- growing annuities and
- perpetuities

I. Single Cash Flows

- A single cash flow is a single cash flow in a specified future time period.
- Cash Flow: CFt

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  Time Period: t

- *The present value of this cash flow is-*

PV of Single Cash Flow = $PV = \left(\frac{CF_t}{(1+r)^t}\right)$

II-Valuing a Stream of Cash Flows

- Valuing a lump sum (single) amount is easy to evaluate because there is one cash flow.
- What do we need to do if there are multiple cash flow?
    - Equal Cash Flows: Annuity or Perpetuity

Unequal/Uneven Cash Flows

$$PV = \frac{FV_1}{(1+i)^1} + \frac{FV_2}{(1+i)^2} + \cdots + \frac{FV_N}{(1+i)^N}$$

$$PV = \sum_{t=1}^{N} \frac{FV_t}{(1+i)^t}$$

Uneven cash flows exist when there are different cash flow streams each year

Treat each cash flow as a Single Sum problem and add the PV amounts together.

III-Annuities

The present value of an annuity is the value now of a series of equal amounts to be received (or paid out) for some specified number of periods in the future.

It is computed by discounting each of the equal periodic amounts.

An annuity is a series of nominally equal payments equally spaced in time.

Present Value of an Annuity

The present value of an annuity can be calculated by taking each cash flow and discounting it back to the present, and adding up the present values.

We can use the principle of value additivity to find the present value of an annuity, by simply summing the present values of each of the components:

$$PV_A = \sum_{t=1}^{N} \frac{Pmt_t}{(1+i)^t} = \frac{Pmt_1}{(1+i)^1} + \frac{Pmt_2}{(1+i)^2} + \cdots + \frac{Pmt_N}{(1+i)^N}$$

- Alternatively, there is a short cut that can be used in the calculation [A = Annuity; r = Discount Rate; n = Number of years]
- Thus, there is no need to take the present value of each cash flow separately
- The closed-form of the PV_A equation is:

$$PV = FV\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right]$$

Annuities Due

- The annuities that we begin their payments at the end of period 1 are referred as *regular annuities (ordinary annuities)*
- An annuity due is the same as a regular annuity, except that its cash flows occur at the beginning of the period rather than at the end

Present Value of an Annuity Due

- The formula for the present value of an annuity due, sometimes referred to as an immediate annuity, is used to calculate a series of periodic payments, or cash flows, that start immediately
- We can find the present value of an annuity due in the same way as we did for a regular annuity, with one exception
- Note from the timeline that, if we ignore the first cash flow, the annuity due looks just like a four-period regular annuity
- Therefore, we can value an annuity due with:

$$PV = CF\left[\frac{1 - \frac{1}{(1+i)^{n-1}}}{i}\right] + CF$$

Deferred Annuities

- A deferred annuity is the same as any other annuity, except that its payments do not begin until some later period
- We can find the present value of a deferred annuity in the same way as any other annuity, with an extra step required

$$PV_{DA} = CF\left[\frac{1-\frac{1}{(1+i)^{n+m}}}{i}\right]-\left[\frac{1-\left(\frac{1}{(1+i)^{m}}\right)}{i}\right]$$

OR

$$PV_{DA} = CF\left[\frac{1-\left(\frac{1}{(1+i)^{n}}\right)}{i}\right]\left[\frac{1}{(1+i)^{m}}\right]$$

Uneven Cash Flows

- Very often an investment offers a stream of cash flows which are not either a lump sum or an annuity
- We can find the present or future value of such a stream by using the principle of value additivity

Present value of a Growing Annuity

- A cash flow that grows at a constant rate for a specified period of time is a growing annuity
- A time line of a growing annuity is as follows

	$A(1+g)$	$A(1+g)^2$	.....	$A(1+g)^n$
0	1	2	......	n

- The present value of a growing annuity can be calculated using the following formula

$$PV_{ga} = A(1+g)\left[\frac{1-\frac{(1+g)^n}{(1+r)^n}}{r-g}\right]$$

- The above formula can be used when
  - The growing rate is less than the discount rate (g<r) or
  - The growing rate is more than the discount rate (g>r)
  - However, it doesn’t work when the growing rate is equal to the discount rate (g=r)

Present value of perpetuity annuity

- A perpetuity is an annuity of with an infinite duration.
- Hence the present value of perpetuity may be expressed as follows

PV_∞ = CF x PVIFA

- Where PV_∞ = present value of a perpetuity
- CF = constant annual cash flows
- PVIFA = present value interest factor for perpetuity (an annuity of infinite duration)

- The value of PVIFA is A perpetuity is an annuity of with an infinite duration.
- Hence the present value of perpetuity may be expressed as follows

PV_∞ = CF x PVIFA

- Where PV_∞ = present value of a perpetuity
- CF = constant annual cash flows
- PVIFA = present value interest factor for perpetuity (an annuity of infinite duration)

The value of PVIFA is $PVIFA = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} = \frac{1}{r}$